Asset pricing in production economies

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Abstract

This paper studies asset returns in different versions of the one-sector real business cycle model. We show that a model with habit formation preferences and capital adjustment costs can explain the historical equity premium and the average risk-free return while replicating the salient business cycle properties. The paper also applies a solution technique that combines loglinear methods with lognormal asset pricing formulae. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

Empirical studies in financial economics have documented important cyclical variations in various security returns and risk premia. In macroeconomics, asset returns have long played an important role as leading economic indicators, and several recent empirical studies emphasize this role.[1] However, little is known, in both cases, about the origins of these relationships. This paper starts filling this gap by studying the question: what version of the standard real business cycle model can explain not only business cycles but also asset market facts, in particular, the puzzling 'equity premium'?

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[1] Some examples of these studies are: Fama and French (1989), Fama (1990), Chen (1991), Estrella and Hardouvelis (1991), and Stock and Watson (1989).

One line of progress for solving the equity premium puzzle has been to modify preferences and payout structures for the case where consumption is specified so as to replicate aggregate data. Most of these studies use the endowment economy framework.[2] However, attempts to explain the equity premium in models with nontrivial production sectors, that is, models where consumption and dividends have to be derived endogenously, were less successful (e.g. Danthine et al., 1992; Rouwenhorst, 1995).[3] To some extent, it should not be too surprising that the difficulty for a general equilibrium model to explain asset returns is increased when consumption and dividends also have to be derived endogenously. For instance, Rouwenhorst (1995) finds that it is more difficult to explain substantial risk premia because endogenous consumption becomes even smoother as risk aversion is increased. The reason behind this is that in the standard one-sector model agents can easily alter their production plans to reduce fluctuations in consumption. This suggests that the frictionless and instantaneous adjustment of the capital stock is a major weakness in this framework. One way to reduce consumption smoothing through the production sector, is to introduce capital adjustment costs. Capital adjustment costs have a long tradition in the investment literature, they also provide a formal framework for the popular 'q' theory (q is defined as the value of the capital stock divided by its replacement cost). It therefore seems natural to introduce capital adjustment costs into this standard framework. In fact, without capital adjustment costs, as most current business cycle models are, these models are plagued by a counterfactual constant q.

Given its previous success in solving the equity premium puzzle in models with trivial production sectors (e.g. Abel, 1990; Constantinides, 1990) our analysis also includes habit formation preferences, in addition to the standard time-separable specification. We can thus study how these preferences fare in general equilibrium when required to jointly explain asset returns and business cycles.

We find that a real business cycle model – by replicating the basic business cycle facts – can generate the historical equity premium with *both* capital adjustment costs and habit formation, but not with either taken separately. The main reason why this combination is successful is quite intuitive: with no habit formation, marginal rates of substitutions are not very volatile, since people do not care very much about consumption volatility; with no adjustment costs, they choose consumption streams to get rid of volatility of marginal rates of substitution. They have to both care, and be prevented from doing anything

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[2] An incomplete list of studies that propose solutions to the equity premium puzzle is: Benninga and Protopapadakis (1990), Abel (1990), Constantinides (1990), Rietz (1988).

[3] Cochrane (1991) presents another approach for intertemporal asset pricing. He evaluates asset pricing relationships derived from producers' first-order conditions.

about it.[4] We develop this point by using a solution method that combines lognormal pricing formulae with a loglinear system for macroeconomic variables. With this method we are able to combine the convenience of linear solution techniques with the ability to study risk premia.

An additional finding of our paper is that while simple habit formation preferences can explain high equity premia, they also generate counterfactually high risk premia for long-term real bonds. Adding financial leverage improves the model's prediction by increasing the equity premium relative to the risk premium for long-term real bonds, and thus helps explain why, historically, stocks have had substantially higher returns than long-term bonds.

The organization of the paper is as follows. Section 2 presents a decentralized equilibrium of our model, Section 3 discusses the model solution, and Section 4 presents results and discusses intuition. Section 5 concludes and contains directions for future work. The appendix presents an analysis of the accuracy of the loglinear approximation method.

2. The model

Most business cycle studies do not consider prices and, with reference to the second welfare theorem, can simply analyze representative agent economies as a planner's welfare maximization. Our interest in asset returns forces us to spell out a decentralized economy in order to explicitly define the securities we are pricing.

Consider the standard real business cycle model with a large number of infinitely-lived firms and households. There is a single consumption–investment good that is produced with a constant-returns-to-scale production technology subject to random shocks in productivity.

2.1. Firms

In each period, the representative firm has to decide how much labor to hire and how much to invest. Managers maximize the value of the firm to its owners (the representative agent) which is equal to the present discounted value of all current and future expected cash flows:

$$\mathrm{E}_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} (A_{t+k} F(K_{t+k}, X_{t+k} N_{t+k}) - W_{t+k} N_{t+k} - I_{t+k}), \tag{2.1}$$

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[4] Since this paper was first circulated, related studies by Boldrin et al. (1995), Danthine and Donaldson (1994), and Tallarini (1994) have confirmed the importance of capital frictions for explaining asset prices in various dynamic general equilibrium models.

with $\beta^k \Lambda_{t+k}/\Lambda_t$ the marginal rate of substitution of the owners. A is the stochastic productivity level, K is the capital stock, N is the quantity of labor input, W is the wage rate, X is the deterministic trend in labor augmenting technical change, and I is investment. The firm's capital stock obeys an intertemporal accumulation equation with adjustment costs:

$$K_{t+1} = (1 - \delta)K_t + \phi(I_t/K_t) K_t, \tag{2.2}$$

where δ is the depreciation rate and $\phi(.)$ a positive, concave function. Concavity of the function, $\phi(.)$, captures the idea that changing the capital stock rapidly is more costly than changing it slowly. This specification also allows the shadow price of installed capital to diverge from the price of an additional unit of capital, i.e., it permits variation in Tobin's q.[5]

The firm does not issue new shares and finances its capital stock solely through retained earnings.[6] The dividends to shareholders are then equal to:

$$D_t = A_t F(K_t, X_t N_t) - W_t N_t - I_t. \tag{2.3}$$

2.2. Households

Representative investors maximize expected lifetime utility of consumption, subject to a sequential budget constraint:

$$\max E_t \sum_{k=0}^{\infty} \beta^k u(C_{t+k})$$

$$\text{s.t. } W_t N_t^s + a_t'(V_t^a + D_t^a) = C_t + a_{t+1}' V_t^a. \tag{2.4}$$

Here a_t is a vector of financial assets held at t and chosen at $t-1$, V^a and D^a are vectors of asset prices and current period payouts. The asset vector a contains shares of the representative firm and possibly other assets. In addition, investors face a (normalized) time constraint $1 = N_t + L_t$, with L representing leisure and N productive work. Given that leisure does not enter the utility function, agents will allocate their entire time endowment to productive work.

For preferences, we consider the standard time-separable case and a simple version of habit formation. The two utility functions can be written as $\Psi_t^{1-\tau}/(1-\tau)$, where Ψ_t is a subutility aggregator. We have then $\Psi_t = C_t$ in the time-separable case and $\Psi_t = C_t - \alpha C_{t-1}$ for habit formation.

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[5] Capital adjustment costs have been studied by Eisner and Strotz (1963), Lucas and Prescott (1971) and Hayashi (1982). Baxter and Crucini (1993) have applied this specification to an open economy real business cycle model.

[6] We consider financial leverage in Section 4.2.3.

2.3. Market equilibrium

In equilibrium, all produced goods are either consumed or invested:

$$A_t F(K_t, X_t N_t) = C_t + I_t,$$

labor supply also equals labor demand. Financial market equilibrium requires that the investors hold all outstanding equity shares and that all other assets are in zero net supply.

3. Model solution and loglinear asset pricing

Prior studies of asset returns in general equilibrium models, like for example Danthine et al. (1992) and Rouwenhorst (1995), use solution techniques that require iterations on nonlinear functions. On the other hand, most studies of macroeconomic quantities in general equilibrium models work with linearizations. Linearization methods are easy and fast to use, however, linearization generally implies that ex-ante returns are equal across securities, which would disqualify such a method for studying risk premia. For our study, we will combine a linearization approach with nonlinear pricing formulae and we will thus be able to address the issue of different ex-ante returns across securities. This method has one main advantage over the conventional way of solving for asset returns that require iterations on nonlinear functions: maintaining some linearization makes this technique easy and fast to use. It is for instance straightforward to handle a model with as many as 50 state variables, something that is generally impossible with purely nonlinear methods.[7]

3.1. A loglinear–lognormal environment

The solution has two basic steps: first we solve for the approximate dynamics of the model, second we apply loglinear pricing formulae. For the first step, we solve for macroeconomic variables using the method outlined by King et al. (1988). This method involves loglinearizing first-order conditions and then solving a linear dynamic system. The solution of the model economy can be represented by a loglinear state space system, with the vector of state variables, s_t, following a first-order autoregressive process with multivariate normal i.i.d. impulses:

$$s_t = M s_{t-1} + \varepsilon_t, \tag{3.1}$$

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[7] A more detailed presentation of this solution method is in Jermann (1994).

where the square matrix M governs the dynamics of the system.[8] For instance, for the economy considered in this paper, s_t contains the capital stock, the productivity level and the habit level. For asset pricing, this system provides us with the log of dividends, d, and the log of the marginal utility, λ, as linear combinations of the state vector.

The second step of our solution method is to apply lognormal pricing formulae following Hansen and Singleton (1983), Campbell (1986) and Campbell (1996). The basic asset pricing formula uses the fact that any claim to a potentially random future payout $D_{t+k}(s_{t+k})$ (for dividend) can be valued by the present value relationship:

$$V_t^{D_k}(s_t) = \frac{\beta^k E_t[\Lambda_{t+k}(s_{t+k})D_{t+k}(s_{t+k})]}{\Lambda_t(s_t)}, \tag{3.2}$$

where β is the pure time discount factor, and $\Lambda_{t+k}(s_{t+k})$ the marginal valuation (or marginal utility) of the numeraire at $t + k$. By assuming that Λ and D are conditionally lognormal, with joint distribution given by Eq. (3.1), we will be able to obtain closed-form solutions for the Euler equation Eq. (3.2). We will also be able to get closed-form solutions for first and second moments of returns for 'strips', that is, assets with a single period payout like in Eq. (3.2). For instance, for our model, the unconditional mean (gross) risk-free rate that we report below, that is, the return on a one-period bond, can be shown to be equal to

$$E(R_{t,t+1}^1(s_t)) = \frac{1}{\beta\gamma^{-\tau}} \exp\left(\frac{1}{2}(\mathrm{var}(E_t\lambda_{t+1} - \lambda_t) - \mathrm{var}(\lambda_{t+1} - E_t\lambda_{t+1}))\right),$$

where γ is the trend growth rate. This expression can be computed analytically from the linear model solution. However, we cannot get closed-forms for unconditional return moments for multiple payout securities such as stocks and coupon bonds. For instance, the (gross) return to the firm's equity, a claim to the infinite sequence of dividends $\{D_{t+k}\}_{k=1}^{\infty}$, defined as

$$R_{t,t+1}^D(s_{t+1}) = \frac{V_{t+1}^D(s_{t+1}) + D_{t+1}(s_{t+1})}{V_t^D(s_t)},$$

can easily be computed from the model's linear solution. However, we need simulations to find the unconditional mean. As shown by Jermann (1994), a notable feature of this method is that it does not impose constant risk premia

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[8] We require the system to be stationary or $I(1)$ nonstationary. Thus, the characteristic roots of M have modulus less than or equal to one. Remember also that a VAR of any order can always be rewritten as a first-order system. A deterministic trend can be incorporated in the way outlined by King et al. (1988).

for multiple payout securities unlike other lognormal methods such as Hansen and Singleton (1983). We provide some examples of the accuracy of this solution method in the appendix.

4. Quantitative model predictions

The objective of the quantitative evaluation is to examine different model versions with respect to their ability to explain the historic equity premium and the average risk-free rate while replicating the salient business cycle statistics such as output, consumption and investment volatility. The calibration is carried out in 2 steps. A first set of parameters is chosen based on National Income Account data, following the standard in the business cycle literature. A second set of parameters, for which precise *a priori* knowledge is weak, is chosen to maximize the model's ability to replicate a set of business cycle and asset pricing moments. In addition to this benchmark parameterization, a detailed sensitivity analysis provides us insights about the model mechanisms at work.

4.1. Calibration

4.1.1. Long-run behavior
A first set of parameters is chosen to match long-run model behavior. These parameters do not significantly affect model dynamics and we use standard values. The quarterly trend growth rate is 1.005, the capital depreciation rate δ is 0.025, the constant labor share in a Cobb–Douglas production function is 0.64.[9]

4.1.2. Productivity shocks
Estimates of the Solow residuals A_t, typically yield a highly persistent AR(1) process in levels.[10] We pick the standard deviation of the shock innovation such as to replicate US postwar quarterly output growth volatility of 1%. Given that output in the model closely mimics the productivity process, output in all calibrated model versions matches this standard deviation. For the persistence parameter of the AR(1) we will choose a benchmark value between 0.95 and 1, as explained below, and present some sensitivity analysis.

4.1.3. Power parameter in the utility function
The power parameter in the utility function, τ, corresponds to risk aversion in the time-separable case. For the non-time-separable specification, this exact

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[9] These are the parameters chosen by Kydland and Prescott (1982); see also King et al. (1988) for details about how to calibrate long-run model behavior.

[10] See Prescott (1986) for a discussion of Solow residuals' estimates.

relationship breaks down, although τ is still related to risk aversion.[11] To be able to primarily focus our study on the effects of habit formation preferences, we will fix τ at 5 for our benchmark parameterization. As we discuss further below, the conclusions of this study do not hinge on this choice.

4.1.4. Habit formation, capital adjustment costs, time-preference and shock persistence

Empirical studies do not offer much precise guidance when it comes to calibrating habit formation, capital adjustment costs, the pure time discount factor and the shock persistence.[12] For this reason, and given the central role played by these parameters for business cycles and asset returns, we pick them – in a reasonable range – to maximize the model's ability to match a set of moments of interest. Let θ_1 denote the vector of the 4 model parameters: $\theta_1 = [\alpha, \beta, \xi, \rho]'$. ξ stands for the elasticity of the investment capital ratio with respect to Tobin's q, the only parameter we need to specify for the capital adjustment costs technology. The parameter ρ stands for the shock persistence. We will choose θ_1 in order to minimize $\Im = [\theta_2 - f(\theta_1)]' \Omega [\theta_2 - f(\theta_1)]$, where θ_2 is the vector of moments to match, $f(\theta_1)$ contains the corresponding moments generated by the model, and Ω is the weighting matrix.

The set of moments to match, θ_2, is obtained from historical business cycle and return data. These are: (1) the standard deviation of consumption growth divided by the standard deviation of output growth, (2) the standard deviation of investment growth divided by the standard deviation of output growth, (3) the mean risk-free rate, and (4) the equity premium.

Practically, we compute $\Im$ for a grid of values for θ_1: $\alpha = [0:0.9]$, $\beta^* = \beta\gamma^{1-\tau} = [0:0.999]$, $\xi = [0.16:\infty]$, and $\rho = [0.95:1]$. Given our model solution procedure, the first three corresponding model (population) moments, in $f(\theta_1)$, can simply and quickly be computed from the model's decision rules. For the equity premium, however, we do not have a closed-form solution and thus we obtain the equity premium by taking the average over 100 simulations each 200 quarters long. For the following parameter values we can

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[11] For detailed discussions of this issue see Campbell and Cochrane (1995), Ferson and Constantinides (1991) and Jermann (1994).

[12] Selecting their parameter values informally, Constantinides (1990) uses a habit formation level of $\alpha = 0.8$, Cochrane and Hansen (1992) use 0.5 and 0.6. To specify the capital adjustment cost technology we need to specify only one single parameter given our solution method: the elasticity of the investment capital ratio with respect to Tobin's q. Abel (1980) estimated this parameter in a somewhat different model and obtained values between 0.27 and 0.52.

drive down $\mathfrak{I}$ to 0.00001:

$$\alpha = 0.82, \quad \beta^* = 0.99, \quad \xi = 0.23, \quad \rho = 0.99.^{13}$$

4.2. Results

We find that our benchmark model with habit formation and capital adjust-ment costs is able to match the selected business cycle and asset return statistics. However, neither a model with habit formation nor a model with capital adjustment costs *alone*, is able to explain the equity premium. As shown by Table 1, the model can generate historical consumption, investment (and out-put) volatility and average historical returns for risk-free bonds and equity. In contrast, the standard RBC model has an equity premium of 2 basis points, and even when increasing risk aversion to 10, the premium is still only 26 basis points annually.[14]

Table 1 demonstrates that adding habit formation preferences alone to the standard RBC model cannot explain large equity premia. Although these preferences have shown to be very effective in previous studies in generating large equity premia, they fail here when consumption and dividends are endo-genously determined. For proof, the equity premium is a mere 3 basis points in our benchmark parameterization without capital adjustment costs. The reason for this blatant failure is the following: once agents are allowed to endogenously chose their consumption paths – and given habit preferences that feature strong aversion for intertemporal substitution – they end up choosing very smooth consumption streams. For example, in the standard RBC model consumption volatility is at 0.77, with habit formation, consumption volatility has dropped to 0.33. Basically, the agents in this economy use the near linear technology to smooth out consumption, counteracting the effect on risk premia habit has on the preference side.

Capital adjustment costs make it more costly to smooth consumption through changing the capital stock, therefore consumers end up taking more consumption risk. As Table 1 shows, this can generate an equity premium of

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[13] Given that we can basically drive down the criterion $\mathfrak{I}$ to zero, the choice of the weighting matrix is to some extent irrelevant, we use the identity matrix for our practical purpose. For values of the power parameter different from 5 the 'benchmark' parameterization would be different. How-ever, our explorations suggest that the selected set of moments can still be replicated. Indeed, the power parameter and habit formation are to some extent substitutes in generating consumption curvature, thus, roughly speaking, with a lower power parameter habit will have to be stronger and with a higher power parameter habit will have to be weaker.

[14] The equity premium is defined, as is customary, as the difference between the unconditional mean equity return and the unconditional mean risk-free rate. All returns are annualized, business cycle data is for quarterly frequency.

Table 1
Business cycles and asset returns

Model version/Moments	$\sigma_{\Delta C}/\sigma_{\Delta Y}$	$\sigma_{\Delta I}/\sigma_{\Delta Y}$	$E(r^f)$	$E(r^e - r^f)$	$Std(r^f)$	$Std(r^e)$	$E(r^b - r^f)$
Benchmark	0.49	2.64	0.82	6.18	11.46	19.86	5.69
Standard RBC model (No habit, no adj. costs)	0.77	1.54	4.26	0.02	0.62	1.02	0.04
Risk aversion = 10, no habit, no adj. costs	0.78	1.53	3.36	0.26	0.76	2.90	0.29
Habit, no adjustment costs	0.33	3.00	4.20	0.03	0.59	1.21	0.08
Adjustment cost, no habit	1.14	0.68	3.91	0.67	0.61	6.09	0.45
Random walk productivity	0.55	2.57	0.03	6.39	11.98	18.80	5.09
Data	0.51	2.65	0.80	6.18	5.67	16.54	1.70

The symbols have the following meaning: $\sigma_{\Delta Y}$, standard deviation of quarterly output growth rates; $\sigma_{\Delta C}$, standard deviation of quarterly consumption growth rates; $\sigma_{\Delta I}$, standard deviation of quarterly investment growth rates; r^f, risk-free interest rate; r^e, return to equity; r^b, return to a perpetual bond in model, long-term government bond in the data. Business cycle growth rate data is from the NIPA, 54.1–89.2, GNP for output, Consumption of nondurables and services for consumption, Fixed investment for investment. Equity and short-term bond returns are from Mehra and Prescott (1985) long term government bond returns are from Ibbotson (1994). Business cycle data is quarterly and asset return data is annualized, both are in percentage terms. Business cycle data and risk-free rates are (computed) population moments, the remaining asset return moments are averages of 100 simulations each 200 periods long.

6.18 in the benchmark model version. Despite this success, it is clear that capital adjustment costs alone do not get past the lack of curvature problem of the standard time-separable preferences – only combined with habit formation can we get the historical equity premium.

4.2.1. Intuition: Where does the equity premium come from?

Additional insights into where the equity premium comes from can be obtained from the model's impulse response functions. For this reason, we present here a decomposition of the equity premium that allows us to directly relate impulse responses to risk premia. This decomposition will put more light on the issue of why capital adjustment costs help create a large equity premium.

Consider the risk premium for holding a security that promises a stochastic dividend k-periods from now: a k-period dividend strip. Given that a stock is just a claim to an infinite sequence of dividends, the equity premium will just be a composite of such strip premia. With some algebra, the following strip premium in our loglinear framework can be written as:

$$E_t(R^{D_k}_{t,t+1})/R^1_{t,t+1} = \exp(-\text{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k}))$$
$$\times \exp(-\text{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1})), \quad (4.1)$$

with $E_t(R^{D_k}_{t,t+1})$ the expected strip return, and $R^1_{t,t+1}$ the risk-free interest rate. Eq. (4.1) shows that the conditional expected excess return of a claim to D_{t+k} can usefully be divided into two components: a *payout uncertainty premium*, the first covariance term and a *term premium*, the second covariance term.[15]

The first element of Eq. (4.1), the premium for payout uncertainty,

$$-\text{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k}), \quad (4.2)$$

is linked to a possible capital gain or loss at time $t + 1$.[16] Indeed, if capital gains (an upward revision of expected payouts, $E_{t+1}d_{t+k}$) are negatively correlated with the valuation, λ_{t+1}, then a risk premium is needed to compensate the investor for the undesirable cyclical property of this asset. This is the type of covariance we usually think as representing the equity risk premium, however, the strip premium has a second component. The second element of Eq. (4.1),

$$-\text{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1}), \quad (4.3)$$

can be thought of as a holding or term premium for a k-period discount bond that depends on the term structure of interest rates as of $t + 1$. For example, if

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[15] This decomposition has first been applied by Campbell (1986) to a consumption endowment model. Jermann (1994) contains additional results about returns to strips and multiperiod-payout securities in the loglinear–lognormal framework.

[16] For $k = 1$, instead of a capital gain or loss we have a positive or negative dividend innovation.
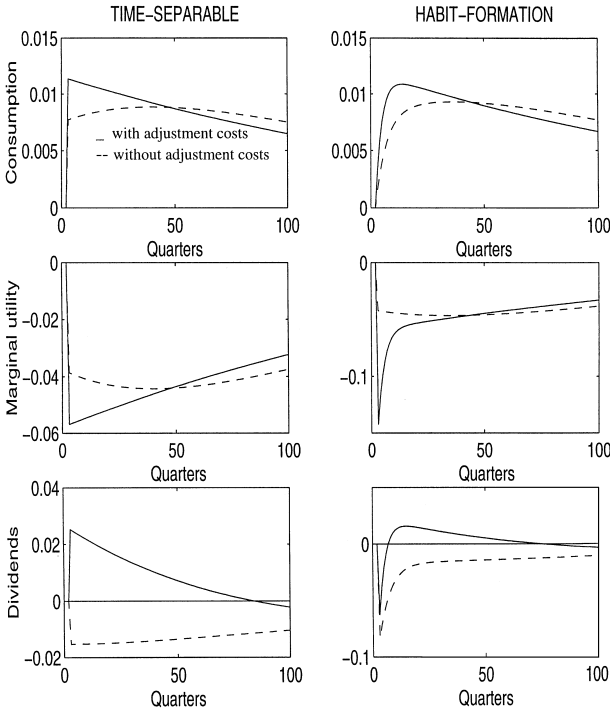
The impulse is a 1% positive productivity shock, the responses are in % deviations from steady state values.

Fig. 1.   Impulse responses for consumption, marginal utility and dividends.

capital gains – a consequence of lower interest rates in $t + 1$ (this is the case when $E_{t+1}\lambda_{t+k}$ exceeds λ_{t+1}) – are contemporaneous with low valuation, λ_{t+1}, then this unfavorable correlation has to be compensated by a positive risk premium. To summarize, this second piece of the risk premium compensates the investor for the uncertainty attached to the valuation of a given payout, whereas the first piece is compensation for the uncertainty over the amount of the payout.

We can now take a closer look at model responses to productivity shocks. The two top panels in Fig. 1 show consumption responses to a 1% positive productivity impulse. For both preference specifications, time-separable and habit-formation, consumption becomes less smooth with capital adjustment costs.[17] More specifically, we should look at marginal utility, λ, in the second row of

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[17] For a detailed analysis of the serial correlation properties of consumption with habit formation see Heaton (1993).

Fig. 1.[18] The picture shows that with capital adjustment costs both parts of the covariance expressing the term premium in Eq. (4.3) contribute positively. First, marginal utility jumps more at impact, and second, it is negatively serially correlated, that is, $E_{t+1}\lambda_{t+k}$ exceeds λ_{t+1}. And as the picture on the right side shows, this second effect is particularly strong with habit formation.

The bottom panels in Fig. 1 display dividend impulse responses in the two models. With and without habit, dividends are more procyclical with capital adjustment costs, without capital adjustment costs, they even turn out to be countercyclical. Thus, a claim to the firms dividend is generally more risky than a perpetual bond in the model with capital adjustment cost and less risky than a perpetual bond in the model without capital adjustment costs. This impulse response is reflected in the size of the equity premium versus the premium for long-term bonds, where we define long-term bonds in the model to be a perpetuity. For instance, Table 1 shows that the equity premium is larger than the bond premium for the time-separable model with capital adjustment costs, 0.67 compared to 0.45. By contrast, in the model without capital adjustment costs, the bond premium, at 0.04, is larger than the equity premium at 0.02. This impulse response is explained by again considering a positive productivity shock. Clearly, capital income (output minus labor income) increases.[19] However, increased investment by the firm – particularly in the near term – tends to reduce the dividend payouts, and thus generate countercyclical payouts. With convex capital adjustment costs, the investment/capital ratio deviates less from its steady-state level and this tends to lead to more procyclical dividends which creates larger premia for payout uncertainty.

4.2.2. Some problems with habit formation

Despite the obvious success of the model, there is also a major shortcoming. Habit formation preferences display strong aversion to intertemporal substitution, and with the impediment to such substitution from adjustment costs, interest rates vary a lot. Indeed, the benchmark model's interest rate standard deviation of 11.46% is not only close to 20 times higher than the benchmark RBC model, but – more relevantly – also more volatile than the historical 5.67%. A closely related problem comes from the equity premium being very much the same as the premium for long-term bonds, whereas historically, long-term bonds have had much smaller excess returns. Specifically, the

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[18] For the time-separable case we have that $\lambda_t = -\tau c_t$, whereas with habit formation

$$\lambda_t = \frac{\alpha\tau}{(1-\alpha)(1-\alpha\beta)}c_{t-1} - \frac{(1+\alpha^2\beta)\tau}{(1-\alpha)(1-\alpha\beta)}c_t + \frac{\alpha\beta\tau}{(1-\alpha)(1-\alpha\beta)}E_t c_{t+1}.$$

[19] Remember that with a Cobb–Douglas production function income shares for capital and labor are constant. As a consequence, capital income and aggregate output are perfectly correlated.

benchmark model displays a premium for long-term bonds of 5.69%, this premium was historically only at 1.70%. Large bond premia are related to overly volatile interest rates because term premia are compensation for (real) interest rate risk.

Table 1 suggests one way to increase the spread between the equity premium and the premium for long-term bonds: permanent productivity shocks. With an equity premium of 6.39 for the random walk case and a premium for long-term bonds at 5.09, the spread between the two is more than twice the size of the benchmark case. Intuitively, permanent changes in dividends make firms more risky and are reflected in a higher equity premium relative to the corresponding long-term bond premium. Unfortunately, this seems to be only a partial improvement given that short-term interest rates are even more volatile than for the benchmark case.

4.2.3. Financial leverage and the equity premium

Financial leverage seems to be another potential way to improve the model's performance by increasing the equity premium relative to the premium for long-term bonds. We explore this possibility in the following paragraphs.

Without a widely recognized theory for corporate finance at hand, we choose to postulate a set of financial policies that combines simplicity with flexibility. Assume that each period, the firm issues j-period discount bonds for a fixed fraction v/j of its capital stock, K_{t+1}, and pays back its debt that comes to maturity, $(v/j)K_{t-(j-1)}$. The dividends to shareholders are now equal to

$$D_t = A_t F(K_t, X_t N_t) - W_t N_t - I_t - \frac{v}{j}(K_{t-(j-1)} - V_t^{1j} K_{t+1}) \qquad (4.4)$$

with V_t^{1j} the price of a j-period discount bond. The first part of this dividend, $A_t F(K_t, X_t N_t) - W_t N_t - I_t$, is the operating cash flow, whereas the second part, $(v/j)(K_{t-(j-1)} - V_t^{1j} K_{t+1})$, is related to the firm's financial policy. We choose this financial policy because it allows the firm to adjust its debt level smoothly through time as the firm grows, and because, at any time, the firm can have outstanding debt of j-different maturities.[20]

This form of leverage requires the calibration of two parameters, j and v. We set j equal to 20 and 40, corresponding to debt maturities of up to 5 and 10 yr, respectively. We then chose v, that governs the amount of leverage, by considering historical leverage ratios and dividend volatility. Masulis (1988), reports

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[20] The Modigliani–Miller theorem holds in this framework. This means that the financial policy does not affect total firm value and optimizing decisions. The financial policy does affect, however, the cash flows to stockholders, and as such, it does affect the equity premium. Note, that we do not consider default risk. For our model parameterizations, throughout our simulations, total firm value is always substantially larger than the value of outstanding debt.

Table 2
Financial leverage and dividend volatility

	No leverage	Leverage parameter: $v = 0.4$		Leverage parameter: $v = 0.6$	
		Maturity			
		5 yr	10 yr	5 yr	10 yr
Equity premium	6.18	7.16 [0.30]	6.81 [0.24]	7.90 [0.42]	7.29 [0.37]
Dividend volatility (Data: 13.6)	8.44	27.42	8.37	53.33	17.81

Equity premia are expressed in percents per annum. Dividend volatility is defined as the standard deviation of per annum growth rates. The terms in brackets are average leverage ratios, defined as debt divided by total firm value. Model data is obtained as the average of 100 simulations each 200 periods (quarters) long. Historical dividend volatility is from Cecchetti et al. (1990)

leverage ratios – debt as a fraction of total firm value – for the US over the last sixty years to range from 0.13 to 0.44 for market values and from 0.53 to 0.75 for book values. Table 2 shows that as a consequence of our parameter choices, we are always within that range.

Table 2 shows that leverage uniformly increases the equity premium, adding 0.63–1.72% to the equity premium. The table also reveals that the bottleneck in increasing the equity premium with this form of leverage comes from dividend volatility. Cecchetti et al. (1990) report a standard deviation of annual dividend growth rates in the US of 13.6%. Clearly, financial leverage increases the equity premium but by quickly making dividends much more volatile than in historical data. These findings suggests that one of the challenges for modelling leverage in future work will be to contain dividend volatility while still making dividends more risky.

5. Conclusions

This paper has taken a step towards explaining asset pricing facts in the standard macroeconomic real business cycle model. The challenge came from the fact that in this general equilibrium framework consumption and dividend payouts are generated endogenously. We show that one of the standard recipes for solving the equity premium puzzle, namely, habit formation preferences cannot simply be replicated in this more general framework. Indeed, making consumers form consumption habits, will make them choose consumption and dividend profiles that result in low risk premia. We showed that capital adjustment costs, by reducing the agents ability for consumption smoothing, play

a crucial in explaining the equity premium. A model with habit formation and capital adjustment costs can match the equity premium and the mean risk-free rate by also matching a set of salient business cycle statistics.

We also first applied here a methodology that allows one to evaluate and think about asset prices in a large class of macroeconomic models. Lognormal closed forms for risk premia are related to a loglinear model solution. While being easy and fast to use, we also show in the appendix some examples of the remarkable accuracy of this method.

Finally, even though successful in generating large equity premia the study highlights some of the remaining shortcomings of this model. In particular, risk premia for long-term bonds are too high relative to the equity premium. We provide some quantitative demonstration how leverage can help along this dimension by increasing the riskiness of stocks without affecting term premia. Future work towards increasing the set of moments to be explained could use this as a starting point for introducing a more developed corporate finance building block into the RBC model. An alternative approach towards increasing the set of stylized facts the model can explain would be to explore different versions of non time-separable preferences in the standard RBC model. Campbell and Cochrane (1995) or Abel (1995) are possible candidates from the consumption asset pricing literature.

Acknowledgements

Appendix

The objective of this Appendix is to provide some examples of the accuracy of the employed solution method. We compare quantitative predictions from our loglinear–lognormal framework to a state-of-the-art, fully nonlinear approach that should be very accurate. For the cases examined here, the two methods yield indeed very similar results which we interpret as a sign of the high accuracy of our loglinear–lognormal method.

When solving the model in a fully nonlinear setting we need to specify the capital adjustment cost function – for the loglinear approximation a single

elasticity was enough. This function is specified as

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{b}{1-a}\left(\frac{I_t}{K_t}\right)^{1-a} + c,$$

where the coefficients b and c are set so as to yield the same steady-state properties and where a replicates the elasticity parameter ξ discussed above. Our nonlinear solution method follows Judd (1992). In particular, we use a two-dimensional complete polynomial basis with Chebyshev polynomials of up to the fifth order, and as choice for the projection condition we use the Galerkin method.

One difficulty in trying to evaluate the results from the loglinear approximation is to replicate the exogenous shock distribution. We follow Tauchen (1986) in specifying the first-order autoregressive productivity process with a discrete Markov process. However, we select the autoregressive parameter and the shock variance such as to exactly fit the first-order serial correlation and the

Table 3
Loglinear–lognormal solution compared to fully nonlinear approach

		$\sigma_{\Delta Y}$	$\sigma_{\Delta C}/\sigma_{\Delta Y}$	$\sigma_{\Delta I}/\sigma_{\Delta Y}$	$E(r^f)$	$E(r^e - r^f)$
Standard RBC model	Loglinear	0.01	0.77	1.54	4.26	0.02
(no habit, no adjustment costs)	Nonlinear	0.01	0.77	1.55	4.28 (0.08)	0.003
Adjustment costs, no habit	Loglinear	0.01	1.14	0.68	3.91	0.67
	Nonlinear	0.01	1.14	0.68	3.93 (0.10)	0.67
Habit, no adjustment costs	Loglinear	0.01	0.13	3.22	4.49	0.0047
(shock persistence 0.95)	Nonlinear	0.01	0.13	3.22	4.49 (0.04)	0.0050
Habit, adjustment costs	Loglinear	0.01	0.38	2.77	1.52	5.93
(shock persistence 0.95)	Nonlinear	0.01	0.38	2.77	1.55 (0.10)	5.90

The symbols have the following meaning: $\sigma_{\Delta Y}$, standard deviation of quarterly output growth rates; $\sigma_{\Delta C}$, standard deviation of quarterly consumption growth rates; $\sigma_{\Delta I}$, standard deviation of quarterly investment growth rates; r^f, risk-free interest rate; r^e, return to equity. Business cycle data is quarterly and asset return data is annualized, both are in percentage terms. Loglinear business cycle data and risk-free rates are (computed) population moments, the equity premium is the average over 100 simulations each 200 periods long. Moments for the nonlinear solution are obtained as the average over 100 simulations each 200 periods long for the time-separable cases, and as the average over 1000 simulations each 200 periods long for habit-formation cases. Numbers in parentheses are standard errors across simulations divided by the square root of the number of simulations, no standard errors are reported when they are negligible.

standard deviation of the growth rate of the univariate process. As can be seen in Tauchen (1990), the standard method fits these moments only approximately which can lead to discrepancies when one is interested in asset prices. For the time-separable model we use a discrete Markov process with 35 states, whereas for the habit formation model we limit ourselves to 12 states to contain computational time. Given that a 12-state process does not well approximate the extremely persistent productivity process with serial correlation of 0.99 that we use in the main part of the paper, we consider here a persistence of 0.95 for the habit formation case.

Table 3 presents the moments we are interested in for this study, computed for both solution methods: loglinear–lognormal and fully nonlinear. As is well documented in numerous studies, for instance in Taylor and Uhlig (1990), the loglinear approach is extremely accurate for macroeconomic variables in the standard time-separable case. Our model with habit formation fares equally well. Moreover, our loglinear–lognormal method also clearly does an excellent job for the asset pricing moments considered in this study for the time-separable case and the habit formation case. Although it is hard to generalize these findings to other models and other asset pricing moments, the loglinear–lognormal approach seems a very attractive alternative not just for quantity variables but also for asset returns given the obvious substantial reduction in programming and computing time.

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