Dynare Summer School 2021

Introduction to Dynare - exercise session 1

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For these exercises, you will use the examples contained in `Session1.zip`.

Exercise 1: Stochastic model For this exercise, we use the small scale New Keynesian model from Sungbae An and Frank Schorfheide (2006) *Bayesian Analysis of DSGE models.* Federal Reseerve Bank of Philadelphia, WP No. 06-5, (available in `papers/an_schorfheide_2006.pdf`) and Edward Herbst and Frank Schorfheide (2016) *Bayesian Estimation of DSGE models.* Princeton University Press.

Look at these different representations of the model (refer to the original paper for the meaning of the variables and parameters):

1. Model A, the original nonlinear model contains the following equations:

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right] && (1) \\
1 &= \phi(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] \\
&\quad - \phi \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi) \pi_{t+1} \right] && (2) \\
&\quad + \frac{1}{\nu} \left[1 - \left(\frac{C_t}{A_t} \right)^{\tau} \right] \\
Y_t &= C_t + G_t + AC_t && (3) \\
AC_t &= \frac{\phi}{2} (\pi_t - \pi)^2 Y_t && (4) \\
G_t &= \frac{g_t - 1}{g_t} Y_t && (5) \\
R_t &= R_t^{\star 1 - \rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} && (6) \\
R_t^{\star} &= r \pi^{\star} \left(\frac{\pi_t}{\pi^{\star}} \right)^{\psi_1} \left(\frac{Y_t}{Y_t^{\star}} \right)^{\psi_2} && (7) \\
\ln A_t &= \ln \gamma + \ln A_{t-1} + \ln z_t && (8) \\
\ln z_t &= \rho_z \ln z_{t-1} + \epsilon_{z,t} && (9) \\
\ln g_t &= (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} && (10) \\
Y_t^{\star} &= (1 - \nu)^{\frac{1}{\tau}} A_t g_t && (11)
\end{aligned}
$$

2. Model B is the stationary version of the model. It is obtained by removing the productivity trend from consumption, $c_t = C_t/A_t$, outptut, $y_t = Y_t/A_t$, and, natural output, $y_t^\star = Y_t^\star/A_t$. In addition, we substitute out G_t and AC_t:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{12}$$

$$\begin{aligned} 1 = {} & \phi(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] \\ & - \phi\beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \pi) \pi_{t+1} \right] \\ & + \frac{1}{\nu} \left(1 - c_t^\tau \right) \end{aligned} \tag{13}$$

$$y_t = c_t + \frac{g_t - 1}{g_t} y_t + \frac{\phi}{2} (\pi_t - \pi)^2 y_t \tag{14}$$

$$R_t = R_t^{\star 1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} \tag{15}$$

$$R_t^\star = r\pi^\star \left(\frac{\pi_t}{\pi^\star} \right)^{\psi_1} \left(\frac{y_t}{y_t^\star} \right)^{\psi_2} \tag{16}$$

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t} \tag{17}$$

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} \tag{18}$$

$$y_t^\star = (1 - \nu)^{\frac{1}{\tau}} g_t \tag{19}$$

3. Steady state of the stationary model is

$$\pi = \pi^\star \tag{20}$$

$$r = \frac{\gamma}{\beta} \tag{21}$$

$$R = r\pi^\star \tag{22}$$

$$c = (1 - \nu)^{\frac{1}{\tau}} \tag{23}$$

$$y^\star = gc \tag{24}$$

$$y = y^\star \tag{25}$$

4. Model C is obtained by using variables defined as ratio to the steady state value, $\hat{x}_t = \ln\left(\frac{x_t}{x}\right)$:

$$1 = \mathbb{E}_t\left[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi}_{t+1}}\right] \tag{26}$$

$$\begin{aligned} 0 = &\left(e^{\hat{\pi}_t}-1\right)\left[\left(1-\frac{1}{2\nu}\right)e^{\hat{\pi}_t}+\frac{1}{2\nu}\right] \\ &-\beta\mathbb{E}_t\left[\left(e^{\hat{\pi}_{t+1}}-1\right)e^{-\tau\hat{c}_{t+}+\tau\hat{c}_t+\hat{y}_{t+1}-\hat{y}_t+\hat{\pi}_{t+1}}\right] \\ &+\frac{1-\nu}{\nu\phi\pi^2}\left(1-e^{\tau\hat{c}_t}\right)\end{aligned} \tag{27}$$

$$e^{\hat{c}_t-\hat{y}_t} = e^{-\hat{g}_t}-\frac{\phi\pi^2 g}{2}\left(e^{\hat{\pi}_t}-1\right)^2 \tag{28}$$

$$\begin{aligned}\hat{R}_t = &\rho_R\hat{R}_{t-1}+(1-\rho_R)\psi_1\hat{\pi}_t \\ &+(1-\rho_R)\psi_2(\hat{y}_t-\hat{g}_t)+\epsilon_{R,t}\end{aligned} \tag{29}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\epsilon_{g,t} \tag{30}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1}+\epsilon_{z,t} \tag{31}$$

5. Finally, model D is the log-linearized version of the original model

$$\hat{y}_t = \mathbb{E}_t[\hat{y}_{t+1}]-\frac{1}{\tau}(\hat{R}_t-\mathbb{E}_t[\hat{\pi}(+1)]-\mathbb{E}_t[\hat{z}(+1)]) \tag{32}$$

$$+\hat{g}-\hat{g}(+1) \tag{33}$$

$$\pi_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}]+\kappa(\hat{y}_t-\hat{g}_t) \tag{34}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1}+(1-\rho_R)(\psi_1\hat{\pi}_t+\psi_2(\hat{y}_t-\hat{g}_t))+\epsilon_{R,t} \tag{35}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\epsilon_{g,t} \tag{36}$$

$$\hat{z}_t = \rho_z z_{t-1}+\epsilon_{z,t} \tag{37}$$

with

$$\kappa = \tau\frac{1-\nu}{\nu\pi^2\phi}$$

Questions:

1. Would model A and model B generate the same trajectories for the endogenous variables?

2. Would model B and model C generate the same trajectories for the endogenous variables?

3. Check the transformation of the first equation from its original form, equation (1), to its expression in terms of stationary variables, equation (12).
4. Model B is available in `models/as/as1.mod`. Fill in the missing equation and run `as1.mod` in Dynare.
5. Verify equation (22)
6. Check the transformation of equation (12) to equation (26)
7. Model C is implemented in `models/as/as2.mod` and a first order approximation is computed. Run it and compare the mean of the variables between `as1.mod` and `as2.mod`. Where is the difference coming from?
8. Model D is implemented in `models/as/as3.mod`. Run it and compare the results with `as2.mod`?

Exercise 3: Higher order approximations

1. In `models/jermann98/jermann98.mod`: change the value of the parameters for consumption habits and investment adjustment cost and observe the effect on the risk premium. Try second and third order approximation.

Exercise 4: Linearization

1. Compute the first order Taylor expansion of equation (26)
2. Compute the first order Taylor expansion of equation (28)
3. Combine the first order Taylor expansion of equations (26) and (28) to obtain equation (32)