Solution function

$$y_t = g(y_{t-1}, u_t, \sigma)$$

where σ is the stochastic scale of the model. If $\sigma = 0$, the model is deterministic. For $\sigma > 0$, the model is stochastic.
Under some conditions, the existence of $g()$ function is proven via an implicit function theorem. See H. Jin and K. Judd (2002) "Perturbation methods for general dynamic stochastic models"

(http://web.stanford.edu/~judd/papers/PerturbationMethodRatEx.pdf)

Solution function (continued)

Then,

$$
\begin{aligned}
y_{t+1} &= g(y_t, u_{t+1}, \sigma) \\
&= g(g(y_{t-1}, u_t, \sigma), u_{t+1}, \sigma) \\
F(y_{t-1}, u_t, \epsilon_{t+1}, \sigma) & \\
&= f(g(g(y_{t-1}, u_t, \sigma), \sigma\epsilon_{t+1}, \sigma), g(y_{t-1}, u_t, \sigma), y_{t-1}, u_t)
\end{aligned}
$$

$$
E_t \left\{ F(y_{t-1}, u_t, \epsilon_{t+1}, \sigma) \right\} = 0
$$

The perturbation approach

- Obtain a Taylor expansion of the unkown solution function in the neighborhood of a problem that we know how to solve.
- The problem that we know how to solve is the deterministic steady state.
- One obtains the Taylor expansion of the solution for the Taylor expansion of the original problem.
- One consider two different perturbations:
  1. points in the neighborhood from the steady sate,
  2. from a deterministic model towards a stochastic one (by increasing σ from a zero value).

The perturbation approach (continued)

- The Taylor approximation is taken with respect to y_{t-1}, u_t and σ, the arguments of the solution function

$$y_t = g(y_{t-1}, u_t, \sigma).$$

- At the deterministic steady state, all derivatives are deterministic as well.

Steady state

A deterministic steady state, $\bar{y}$, for the model satisfies

$$f(\bar{y}, \bar{y}, \bar{y}, 0) = 0$$

A model can have several steady states, but only one of them will be used for approximation.
Furthermore,

$$\bar{y} = g(\bar{y}, 0, 0)$$