The general problem

Deterministic, perfect foresight, case:

$$f(y_{t+1}, y_t, y_{t-1}, u_t) = 0$$

Stochastic case:

$$E_t \{f(y_{t+1}, y_t, y_{t-1}, u_t)\} = 0$$

y : vector of endogenous variables

u : vector of exogenous shocks

Solution methods

- For a deterministic, perfect foresight, it is possible to compute numerical trajectories for the endogenous variables
- In a a stochastic framework, the unknown is the decision function:

$$y_t = g(y_{t-1}, u_t)$$

For a large class of DSGE models, DYNARE computes approximated decision rules and transition equations by a perturbation method.

The perturbation approach

- Perturbation approach: recovering a Taylor expansion of the solution function from a Taylor expansion of the original model.
- A first order approximation is nothing else than a standard solution using linearization.
- A first order approximation in terms of the logarithm of the variables provides standard log-linearization.

A general stochastic model

$$E_t \left\{ f(y_{t+1}, y_t, y_{t-1}, u_t) \right\} = 0$$

$$\begin{aligned} E(u_t) &= 0 \\ E(u_t u_t') &= \Sigma_u \\ E(u_t u_\tau') &= 0 \quad t \neq \tau \end{aligned}$$

y : vector of endogenous variables

u : vector of exogenous stochastic shocks

Timing assumptions

$$E_t \{f(y_{t+1}, y_t, y_{t-1}, u_t)\} = 0$$

- shocks u_t are observed at the beginning of period t,
- decisions affecting the current value of the variables y_t, are function of
  - the previous state of the system, y_{t-1},
  - the shocks u_t.

The stochastic scale variable

$$E_t \{f(y_{t+1}, y_t, y_{t-1}, u_t)\} = 0$$

- At period t, the only unknown stochastic variable is y_{t+1}, and, implicitly, u_{t+1}.
- We introduce the *stochastic scale variable*, σ and the auxiliary random variable, ϵ_t, such that

$$u_{t+1} = \sigma \epsilon_{t+1}$$

The stochastic scale variable (continued)

$$E(\epsilon_t) = 0 \tag{1}$$
$$E(\epsilon_t \epsilon_t') = \Sigma_\epsilon \tag{2}$$
$$E(\epsilon_t \epsilon_\tau') = 0 \quad t \neq \tau \tag{3}$$

and

$$\Sigma_u = \sigma^2 \Sigma_\epsilon$$

Remarks

$$E_t \left\{ f(y_{t+1}, y_t, y_{t-1}, u_t) \right\} = 0$$

- The exogenous shocks may appear only at the current period (in the presentation, not in Dynare)
- There is no deterministic exogenous variables
- Not all variables are necessarily present with a lead and a lag
- Generalization to leads and lags on more than one period (nonlinear models require special care for lead terms)